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MATH FUNDAMENTALS FOR AUDIO

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Introduction

What does math have to do with recording music? Audio technology is all about *interconnectivity*, and so is math. As soon as your sound hits the analog-to-digital converter (ADC), every process that runs in the computer is a mathematical one. Wouldn't you like to know what's going on in that box?

I love math, but my strengths are in problem solving, not simple addition. I have to think a little too hard on simple problems—like, Quick! What is 35 plus 7?—or how to split the bill at a restaurant when out with friends.

Still, I love logic problems and making connections, and that is a great thing for an audio engineer. Whether it's routing a digital clock, making connections between an outboard compressor and a console, or getting from a laptop computer's stereo mini jack on stage to a live sound rig at the FOH (front of house): as audio engineers, we have to determine lots of ways to connect devices—and often very quickly!

Connecting mathematical concepts is also fun. I once saw a table of coefficients in a manual describing surround-sound downmixing, and I got excited when I saw the number 0.707. I knew, based on my experiences, that multiplying by 0.707 would reduce whatever input by 3 dB, and I was able to look at the coefficients for each channel and know the resulting attenuation. It was a *Matrix* moment for me, both because of the Hollywood film reference and because it's an actual matrix.

(Left Total) = 1.0L + 0.707C – 0.707S ;
(Right Total) = 1.0L + 0.707C + 0.707S ;

I don't even see the code any more. All I see is left at unity, center +3 dB, surround -3 dB, etc.

In the course of writing this book, I asked a few entertainment industry professionals why math is important. Here are some of my favorite responses:

> "As soon as your audio hits the computer, it is subjected to all sorts of mathematical computations."
>
> —*David Glasser, Airshow Mastering*

"My favorite math equation to share with students is the rate of acceleration due to gravity: 9.8 m/s^2. That's how fast these 2-ton lighting ballasts will fall on you if you don't properly rig them."

—*Daniel Koetting, Associate Professor, University of Colorado Denver*

"When we were building these studios we had to know how to calculate wavelengths. It helps to know where standing waves are, and that the lowest frequencies are the size of this 50-foot long hallway."

—*Mike Cramp, Post Modern Company*

"You have to know percentages when dealing with performing contracts."

—*LeeAnn Weller, Former Events Manager, University of Colorado Denver*

It helps to know that you are actually going to use this stuff during the course of your career after high school and college. Audio engineering and music production are such popular careers right now, and the competition is fierce. Why not give yourself a competitive edge? Being articulate about math will really impress your next client or employer!

Both acoustic sounds and electronic audio have a lot to do with circles and spheres (sound propagation), so you need geometry and trigonometry. Making sounds louder or softer is based on logarithmic addition and subtraction. Digital audio has everything to do with binary numbers, and even calculus is used to describe signal-to-error ratio. And of course, fractions are everywhere.

As an educator in the field of recording arts, I have taught hundreds of students various skills in audio production, including music recording, surround-sound recording, and post-production. I have always been surprised at how intimidated some students become when they have to solve seemingly simple, audio-related math problems. For example:

Problem: How many milliseconds of delay is caused by a plug-in that shows a 4 sample latency (based on a sampling frequency of 48 kHz)?

When solving these kinds of very relevant, everyday problems, a few of my students make mistakes, some do not seem to be trying, and others don't even know where to begin. My conversations with these students reveal that they are excited about learning the math, but have always been challenged by it. It's rare to find students who don't seem to care at all. I love it when students try to help each other out with different ways of thinking about the problem. This is one of the best ways to learn—get help from your friends and think about the problem in a new way.

Whatever challenges you might face as a reader of this text, my aim is simply to show why math is crucial to understanding concepts in audio. Certainly there are various software programs and affordable smartphone apps to solve everyday audio problems: SPL meters, real-time analyzers, room-mode calculators, and more. But somewhere along the way, if you *truly* want to understand how to control and manipulate sound—and if you want to outshine the competition—you should learn some of the math behind the music.

HOW TO USE THIS BOOK

You will the following images throughout this book:

Clue: We found a magic number

When you see this, you'll know that the number will appear again in future sections where I deal with more advanced concepts. "Magic numbers" are based on a lecture I give in which I begin by showing how sine waves relate to RMS (root mean squared) values, and how that in turn relates to signal-to-noise error and its corresponding "6 dB per bit" approximation. Seeing values such as 0.707, 1.414, 3 dB, and 6 dB come up again and again is pretty magical. You'll see why as you read along. The last chapter briefly summarizes these concepts.

More to Know

If you are ready for a few more advanced ideas, continue reading through these sections.

Connecting Components

These indicate sections where you can see how concepts fit together and try your hand at solving exercises.

A WORD ABOUT "MATH ANXIETY"

In her online course "How to Learn Math: For Students," Jo Boaler of Stanford University presents some myths about math.[1] I believe it is important for you to understand that if you have anxiety about learning math, you are not alone. Here are some highlights from her course:

1. It is a myth that math relies on memorization: it does not.
2. Your gender does not predetermine your ability to succeed at math.
3. Keeping a positive attitude is imperative—you can do it!
4. Making mistakes helps you to learn. If you are working your way through a maze, you will bump into walls. If you do the maze a second time, you learn what to avoid.
5. There is more than one way to solve a problem (especially in audio!).

NOTES

1. Jo Boaler, "How to Learn Math: For Students," Stanford Online, 2014, accessed 6 September 2016, https://lagunita.stanford.edu/courses/Education/EDUC115-S/Spring2014/about.

SELF-ASSESSMENT: IS THIS BOOK FOR YOU?

In order to find out whether you'll get any use out of this book, answer "yes" or "no" to the following questions:

Question:	**Yes**	**No**
1. I know a reference for the speed of sound in m/s (meters per second) or ft/s (feet per second). (See pp. 42–43: The speed of sound as a product of wavelength and frequency.)		
2. I know how to find the wavelength of a sine wave if given the frequency. (See pp. 44–45: The speed of sound as a product of wavelength and frequency.)		
3. If I know the beats per minute (bpm) of a song, I can determine how to create an echo or delay (in milliseconds) that matches the tempo. (See p. 55: Converting beats per minute to beats per second.)		
4. If I know the distance from a sound source, I can calculate the delays needed for a speaker array. (See pp. 52–54: A way to find delay times using only distance and speed of sound.)		
5. Given a peak value for a signal, I can find the RMS (root mean squared) value. (See p. 111: Finding the RMS value of a signal.)		
6. Given three or more resistors in parallel, I can find the total resistance represented by those three resistors in a simplified circuit. (See pp. 18–19: Finding the total resistance (RT) for resistors in parallel.)		
7. I can create a polar plot by measuring voltage values recorded from various positions in front of a microphone. (See figure 4-13.)		
8. Given the bandwidth, I can find the Q of an eq curve. (See p. 18: Q and bandwidth.)		
9. Given the impedance and peak voltage, I can find the power output by an amplifier. (See pp. 57–58.)		
10. Given a voltage, I can find a value that is 3 dB lower. (See p. 73: Computing the antilog of a number.)		
11. I know how to determine the signal to noise ratio of a system. (See Chapter 7, "An Advanced Look at Signal-to-Noise Ratio," p. 111.)		
12. I can determine the data rate for five tracks of audio recorded at 48 kHz, 24 bit. (See chapter 1, "Working with Data Rates," p. 34.)		
13. If 3 faders are set to "unity" on an analog console, and each has a sine wave corresponding to 0 VU when soloed, I can predict the VU value when all three are summed together. (See pp. 75–76: "Calculating dB using a VU meter".)		

If you answered “no” to 6 or more questions, you will probably find this book useful.

If you answered “no” to 3–5 questions, you might just want to peruse the more advanced sections.

If you answered “no” to fewer than 2 questions, then you probably do not need this book, but it could be a useful refresher.

ONE
Basic Math Review

USING A CALCULATOR

Let's start with something simple: basic operations on a calculator. You can use your smartphone or computer as long as its calculator has a scientific mode. (Sometimes you can find this by rotating the smartphone's screen so that you are looking at it in "landscape mode.")

Make sure your calculator has the following buttons:

- sin
- cos
- 1/x
- 10^x
- DEG
- RAD
- $\log_{10}$

Finding Logarithms with a Calculator

For example,

Problem: "What is the base 10 logarithmic value of the number 2?"

or, as most people would say,

"what's the log of 2"?

You're definitely going to come across this problem in your audio class. (For more information, see the section on decibels, p. 71.) Fortunately, it's easy. Type the number you want, and then press the "log10" button (Figure 1.1).

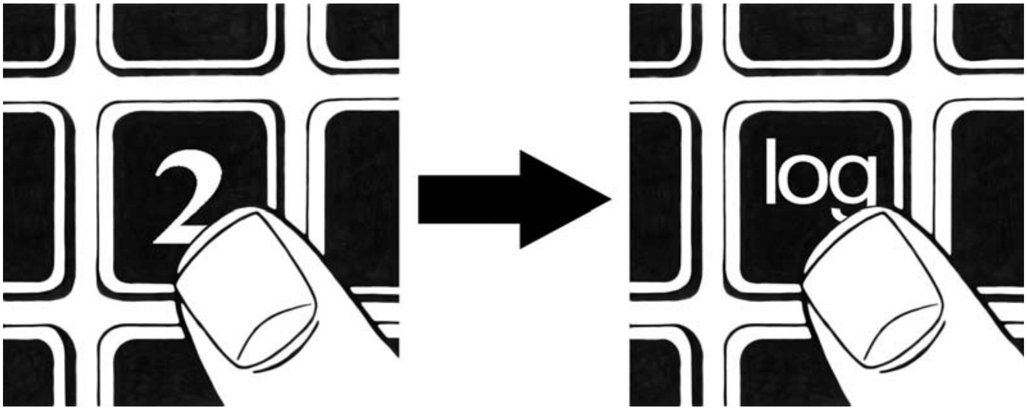

FIGURE 1.1 Calculating the log of 2

The answer is 0.301029995663981. Often, when we get a large number like this, we round the number to three significant digits. In this case, we could shorten this number to 0.301.

Clue—we found a magic number! 0.301

("Clues" are placed around the book when we touch on a concept that relates to magic numbers.)

Finding Trigonometric Values with a Calculator

Likewise, we will be looking at trigonometric functions, such as the sine function. For these functions, we will calculate the value of a point on the curve's axis, and determine where x and y intersect (Figure 1.2). We will go over more in chapter 4.

$$f(x) = \sin(x)$$

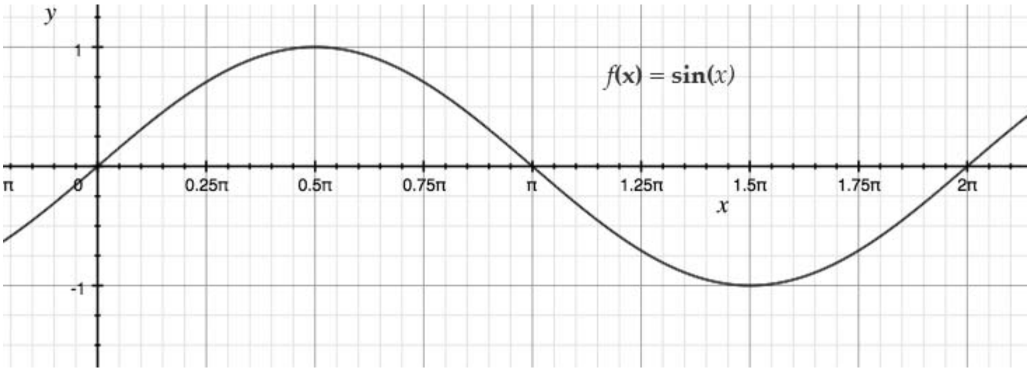

FIGURE 1.2 Graph of $y = \sin(x)$

Depending on how the x-axis is drawn, these functions require us to use either the "RAD" (radians) or "DEG" (degrees) mode on our calculator (Figure 1.3).

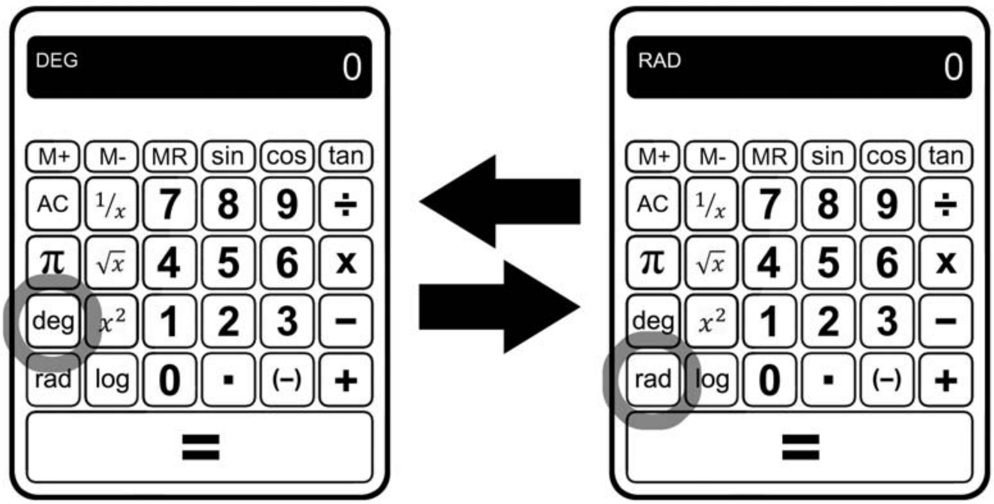

FIGURE 1.3 Switching between radians (a) and degrees (b)